REIMAGINING MUQARNAS:

Exploring Generative Design for Innovative Patterns in Iranian-Islamic Architecture

TARO NARAHARA[1], MOHAMMADMEHDI MOULAII[2], AND MEHDI MOSTAFAVI[3]
[1]*New Jersey Institute of Technology.*
[2,3]*Bu-Ali Sina University, Hamedan, Iran.*
[1]*narahara@njit.edu, 0000-0002-2200-1253*
[2]*m.moulaii@basu.ac.ir, 0000-0003-0079-929X*
[3]*mehdimostafavi0559@gmail.com, 0000-0001-8987-4334*

Abstract. Muqarnas is one of the key identity-forming elements in Iranian-Islamic architecture, serving as both a structural component that fills the transitional zones between walls and domes as well as a decorative element derived from the same structural composition. This paper introduces a method for automatically generating muqarnas patterns parametrically and presents our findings indicating the relationship between subjective perceptions and objective characteristics of muqarnas. Our primary goals are optimizing muqarnas forms and attaining new, better-performing, and more efficient forms. Our analysis results revealed the relationships between design variables of muqarnas patterns, such as the number of facets, the displacement value that indicates structural strength, and the subjective evaluation of their design qualities, such as aesthetics, modernity, traditional quality, and tranquility, using quantifiable measures based on crowdsourcing. By identifying the relationships between the planned forms and their intrinsic attributes in a quantitative data format, this study lays the groundwork for generating new design patterns with specific characteristics.

Keywords. Muqarnas forms, generative design, traditional values.

1. Introduction

Muqarnas is a pivotal architectural embellishment that significantly contributes to the aesthetic enhancement of various Iranian structures, serving diverse purposes. Functioning as a computational design system, muqarnas found application in diverse architectural elements like vaults, domes, arches, and portals, facilitating seamless transitions within buildings. For instance, its prismatic arrangement in portals orchestrates a gradual transformation of architectural plans, creating expansive entrance spaces (Ödekan, 1975). The visual appeal of muqarnas lies in its sculptural

ACCELERATED DESIGN, Proceedings of the 29th International Conference of the Association for Computer-Aided Architectural Design Research in Asia (CAADRIA) 2024, Volume 2, 293-302.

[redacted]

quality, generating captivating light and shadow effects. Figure 1 shows the complex geometry and structure of muqarnas in several religious buildings.

This study aims to contribute to ongoing research on muqarnas geometry/patterns by establishing groundwork for novel designs using algorithmic and parametric techniques. Emphasis will be placed on preserving Islamic architectural heritage while embracing innovative design methodologies. Exploring questions like enhancing muqarnas patterns through generative design and algorithms and preserving their intrinsic value within modern contexts, this research seeks to blend Iranian muqarnas' heritage with generative design frontiers.

2. Related Works

The reconstruction and restoration of muqarnas ornaments have been meticulously analyzed in various academic studies. Dold-Samplonius (1992) laid the foundational groundwork, serving as a cornerstone for Harmsen's algorithmic muqarnas reconstruction in 2006. Harmsen's approach treats the planar representation of muqarnas as a graph, storing three-dimensional data within its nodes. Furthermore, Takahashi (2020) contributed by presenting diverse muqarnas plane projections, including square and pole table styles.

In addition to these analytical studies, design-research investigations have aimed to push the boundaries and explore the creative possibilities of muqarnas geometry. For instance, Yaghan (2001, 2005) introduced new versions of classical muqarnas forms through innovative unit developments. Architect Imani (2017) proposed an alternative workflow for computer-aided muqarnas reconstruction, emphasizing motif-based and layer-based analyses of muqarnas patterns. Similarly, Alaçam, et al. (2017) delved into paper folding techniques to explore muqarnas tectonics conceptually, experimenting with form-finding approaches to understand its generative and structural aspects. Furthermore, Alaçam and Güzelci (2016) explored the potential of parametric modeling software in unlocking abstract muqarnas geometry to generate new designs.

The primary aim of our paper is to revitalize the traditional art of muqarnas by conducting various algorithmic modeling methods. These methods aim to develop software tools for muqarnas modeling, ensuring the preservation of this cultural heritage for future generations. Additionally, modeling muqarnas represents a pivotal step toward a deeper understanding of these intricate geometrical forms.

3. Research Objective

The analysis of literature on 3-D Persian architectural elements, specifically muqarnas, underscores a predominant reliance on 2-D schematics for the examination and reconstruction of patterns despite the inherently three-dimensional nature of these elements. In the context of muqarnas plan projections, the diverse shapes of elements remain discernible; however, the attribution of elements to specific tiers remains elusive within these projections, necessitating a decoding process. Some authors endeavor to incorporate three-dimensional information within 2-D drawings (Gherardini & Leali, 2016).

The primary objective of this project is to employ a framework rooted in fundamental 2-D drawings alongside the utilization of the prevalent muqarnas patterns with various combinations of the parameters and procedural approaches derived from a sample of standard muqarnas patterns. This methodology aims to generate a multitude of muqarnas 3-D patterns based on Islamic 2-D patterns, thereby enabling the analysis and reconstruction of intricate 3-D patterns.

Our objective is to scrutinize various parametrically generated muqarnas patterns to identify combinations of parameters that resonate more closely with, for example, traditional, modern, or sacred designs, according to participant perceptions. Our analysis seeks not only to preserve traditional values but also to align with contemporary and modern architectural styles.

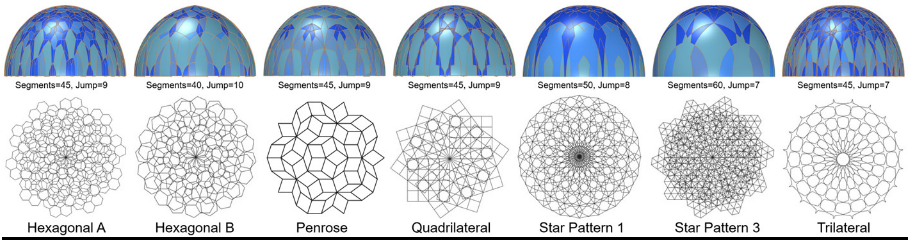
The outcomes of this study will manifest as a virtual, textured model showcasing precise shapes and positional attributes of architectural muqarnas. The proposed framework seeks to provide valuable support to researchers and scholars within the architectural domain by facilitating an in-depth analysis of muqarnas patterns and the underlying geometric principles applied through generative design methodologies.

4. Methods

This research explored the relationships between the subjective and objective values of muqarnas in the contemporary context while preserving traditional values through various methods and analyses aimed at defining, interpreting, and generalizing digital 3-D muqarnas patterns.

The following two methods were proposed in steps. In the first step, we developed a procedural method to generate 60 muqarnas design patterns parametrically. In the second step, we analyzed each of them based on analytically derived objective evaluations and subjective evaluations through crowdsourcing.

The proposed parametric models were implemented using the Rhinoceros software with a procedural approach created using the Grasshopper interface, which facilitates the fast and efficient development of algorithms accessible to non-programmers.



[redacted]

4.1. MUQARNAS DESIGN PATTERN GENERATION

As we discussed in the previous section, 3-D patterns of muqarnas predominantly rely on 2-D schematics of Islamic patterns, so we carefully selected appropriate 2-D shapes for the muqarnas, serving as a fundamental element for subsequent work. The parametric modeling of these base patterns allowed for the creation of variations while maintaining geometric continuity, thereby unveiling the potential of parametric design. The following four steps were performed to generate 60 muqarnas patterns.

Step 1: Seven types of 2-D patterns were carefully selected and developed for this study – Hexagonal A, Hexagonal B, Penrose, Quadrilateral, StarPattern 1, StarPattern 3, and Trilateral (Figure 2). For the initial 2-D pattern generation, we used the Parakeet3D plugin (2023), a powerful tool in Grasshopper for creating intricate designs, and we further modified them to satisfy a wide range of variations ranging from those that resemble traditional muqarnas patterns to those that suggest speculative design patterns, which serve as suitable 2-D bases for parametrically generating various muqarnas patterns.

Step 2: The generation of 3-D muqarnas patterns relied on two primary parameters: segment number and jump number, which were originally defined within the Parakeet plugin. Segment number defines the number of vertices radially around the base area of muqarnas. The jump number defines the region offset ratio from the originally defined 2-D pattern module in Parakeet when tiling them. Two of the authors, who are professionally trained Iranian architects, further finetuned and modified the patterns based on Islamic 2-D shapes. Based on the combinations of two parameters, 60 new muqarnas patterns were generated.

Step 3: Arranging parameters was instrumental in constructing 3-D patterns, leveraging the proportional features of the patterns to generate them while ensuring the creation of 2-D projections that fill the intended area without gaps or overlaps. The numbering sequence that alters the 3-D patterns deviates from the classical tiering system, reflecting the divergence from traditional muqarnas patterns.

Step 4: Morphing a collection of 3-D shapes to a reference dome-like geometry marked the finalization of the muqarnas pattern modeling. This step underscored the potential of the parametric design workflow. These 3-D shapes originated from basic muqarnas 2-D patterns, which are prevalent in Iranian muqarnas. As a result, the patterns derived in this step are similar to muqarnas elements and their fundamental patterns.

4.2. DATASET CREATION AND EVALUATIONS

After generating 60 patterns of muqarnas in 3-D, we conducted a series of evaluations to understand the relationships between parameters that define geometries and their performances, using both analytical results that are objectively defined and perceptive impressions quantified by subjective evaluations through crowdsourcing on each pattern.

4.2.1. Objective Evaluations

Our objective evaluations include calculating the number of facets used and the structural behavior in terms of a maximum displacement in each pattern.

Facets represent individual panel modules or surfaces that compose the muqarnas, often comprising intricate geometries. These facets contribute to the overall aesthetics, structural stability, or load-bearing capabilities of the muqarnas. The number of facets can be interpreted as the number of elements necessary to construct each muqarnas pattern, indirectly indicating labor for assembly and production, which is approximately proportional to the number of facets (the higher the facet count, the higher the construction cost). Also, higher facet counts lead to more visually denser patterns with more panels.

For conducting structural analyses of different muqarnas patterns, we employed Karamba3D (Preisinger, 2013), an interactive, parametric structural engineering tool in Grasshopper based on Finite Element Analysis (FEA). Using parameters such as a gravity load, a muqarnas pattern, boundary element conditions, and interior forces, Karamba3D provides accurate analyses of muqarnas shells. The analysis outputs from Karamba3D include a maximum displacement in millimeters within one pattern, which indicates the stability of muqarnas structures with varying patterns under a consistent loading condition.

4.2.2. Subjective Scores

Crowdsourcing the generated 3-D models involved soliciting participant feedback through six questions to gauge subjective values associated with the muqarnas. For the assessment, 60 muqarnas images were randomly selected, and the following six question statements were formulated for each image:

Q1: It is a traditional muqarnas design.

Q2: Its impression is modern and contemporary.

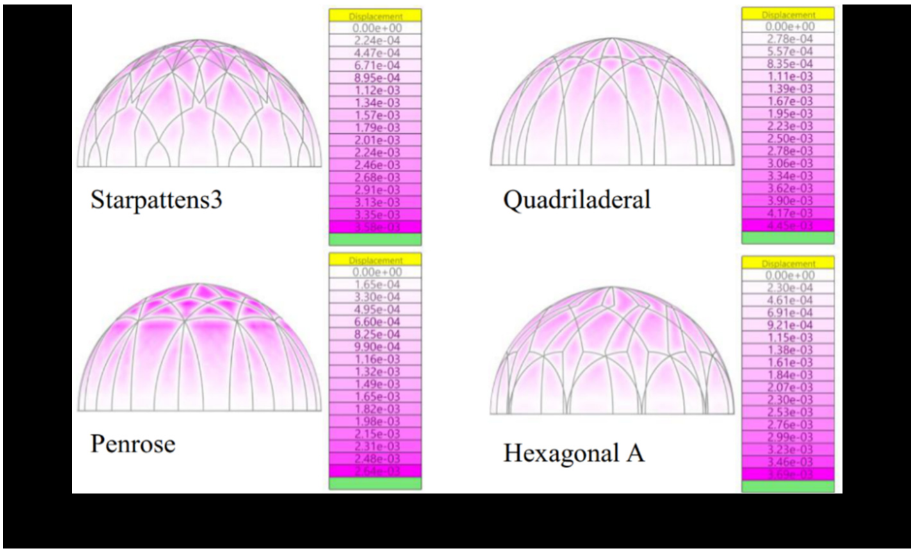
Q3: It is an aesthetically pleasing beautiful design.

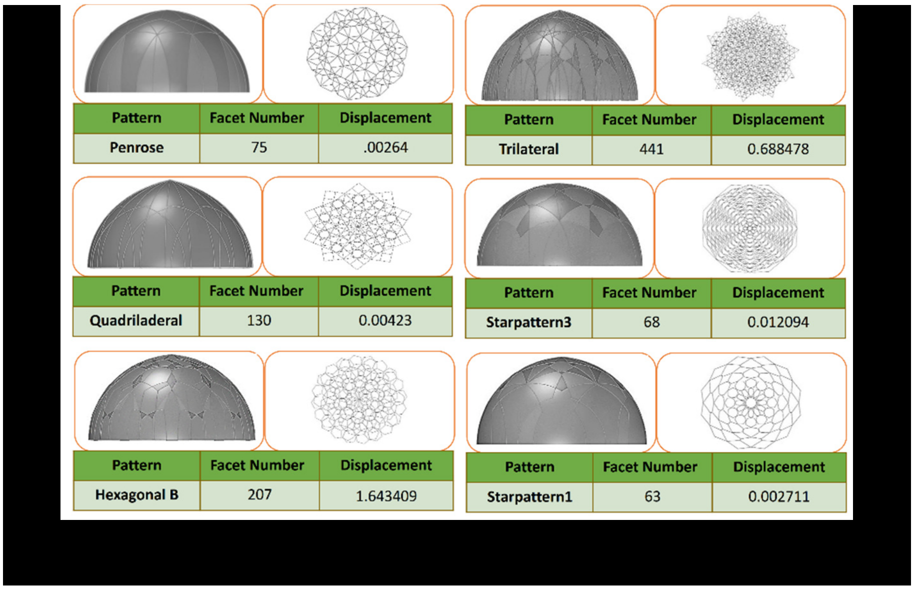
Q4: It is a peaceful, calming, restful design, perfect for meditation.

Q5: It is a sacred design appropriate for a religious purpose.

Q6: Overall, it is personally a preferable design for me.

In total, 25 reviewers participated in this evaluation, all of whom are architects in Iran. This time, we aim to study subjective evaluations by participants who are culturally accustomed to Iranian traditional architectural heritage and patterns, including muqarnas. Each floor plan was evaluated by participants based on a five-grade score on a scale of 1 (strongly disagree) to 5 (strongly agree). Only the muqarnas images were shown to participants for evaluation. A task completion control was implemented,

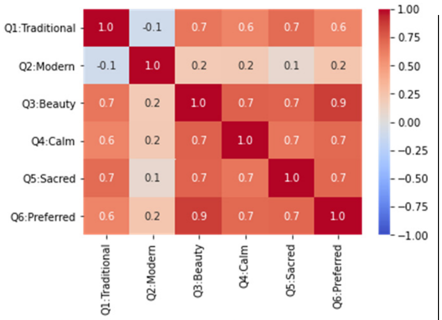


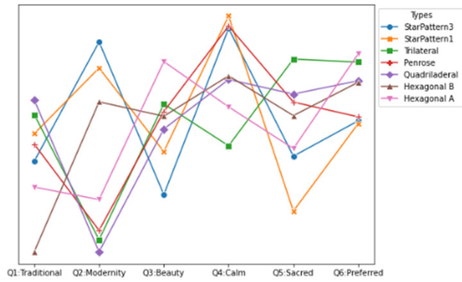


and the participants had to answer all the assigned questions to complete the task; otherwise, their responses were not included in the study. Each participant was asked to evaluate 20 muqarnas images. As a result, each pattern was evaluated by 8 to 11 individuals.

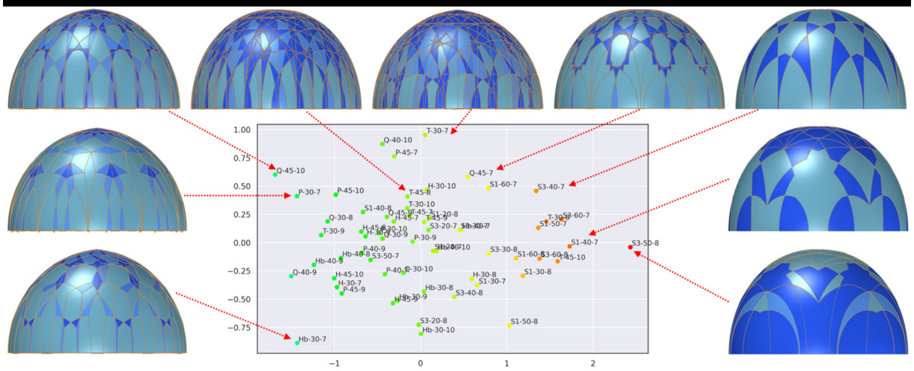
5. Results

The outcomes from questionnaires and objective evaluations, including structural analyses and facet counts, were analyzed to establish a credible basis for discussion and conclusions.





[redacted]



[redacted]

5.1. ANALYSES BASED ON OBJECTIVE EVALUATIONS

Figure 3 shows the displacement values assigned to each of the muqarnas patterns, indicating their deflections when subjected to gravity and internal loads, which are contingent upon the 3-D pattern types, muqarnas supports, and their material composition. A lower displacement value indicates greater stability for the muqarnas, while a higher value denotes less stability. Although influenced by several factors, the analysis unmistakably demonstrates a direct correlation between displacement and the number of facets—more facets in a pattern lead to increased displacement, implying higher stress on the muqarnas structures.

The facet number significantly impacts structural analyses in various ways. Factors such as the facet counts, their shapes, configurations within the pattern, and individual sizes collectively influence the structural analyses of muqarnas. Determining each factor's precise role in increasing or decreasing stress within the muqarnas structure is a complex process. However, as a general observation, 3-D patterns with higher facet counts and smaller facet sizes tend to exhibit increased displacements in muqarnas structures, while the opposite holds true for patterns with fewer facets and larger facet sizes (Figure 4).

5.2. ANALYSES BASED ON SUBJECTIVE EVALUATIONS

After analyzing the responses of all participants to the six questions concerning each pattern, we have arrived at the following results:

As shown in Table 1, besides Q2 (Modernity), our five criteria show strong positive correlations among them. Especially we find the highest correlation between the ratings of Q3 (beauty) and Q6 (preferred) ($r = 0.9$, $p < 0.0001$, as shown in Table 1), suggesting that participants primarily looked at aesthetic value when judging their overall preference for muqarnas patterns. On the other hand, we did not find any correlation between Q2 (Modernity) and other criteria, even including Q1 (traditional). Our study shows that the ratings for modernity and traditional values do not have a simple inverse relationship but rather a more complex one. For example, Hexagonal A demonstrated relatively neutral results with lower scores simultaneously in both Q1 and Q2, yet it was the most overall preferred pattern in Q6 and Q3 (Beauty). Contrary to our general understanding, we found that Q1 (Traditional) and Q2 (Modernity) are not always in an opposite relationship. Furthermore, whether participants' impressions of design patterns are new or old does not influence their overall preference for their design as much as other criteria.

As shown in Figure 5, the Hexagonal A and Trilateral patterns are preferred overall among participants, whereas the Star Patterns 1 and 3 with curved lines exhibit the least popularity (Q6). The preference for the most popular patterns correlates with their abundance of straight lines, traditional facets/shapes within the patterns, and a higher degree of segmentation akin to traditional muqarnas designs. Conversely, the less popular patterns tend to feature curved lines and exhibit fewer segmented forms, deviating from the traditional muqarnas structure, which potentially influences their lower appeal among participants. It is notable that Star Patterns 1 and 3, with the worst overall preferred ratings in Q6, scored the highest in Q2 (Modernity) and Q4 (Calm, Tranquility), defining their unique characteristics.

Figure 6 shows the distribution of 60 muqarnas patterns based on their ratings in six criteria as their features using Principal Component Analysis (PCA). PCA is a dimensionality reduction technique that arranges layouts in a 2-D plot with similar features (performance characteristics) closer together. Similar patterns are plotted closer together, implying that similar geometric patterns perform similarly, as seen from the ID tags of markers such as "Q" (i.e., Quadratic) in Figure 6. Each pattern has strengths and weaknesses. Star-shape-based patterns (S1 & S3) distributed closer to the right side of the plot in Figure 5 have lower overall scores.

5.3. SUMMARY OF THE ANALYSES

In summary, the traditional aspect of muqarnas exhibits positive correlations with various attributes, such as beauty and tranquility, except for modernity. This observation suggests that muqarnas patterns aligned with traditional forms from the past tend to be favored among the participants. Furthermore, it indicates that despite employing virtual generative design techniques, our generated muqarnas patterns retain their traditional values and remain deeply rooted in culturally accepted muqarnas aesthetics.

According to participant responses, the overall preferred muqarnas patterns exhibit moderately high correlations with both the facet number and their geometry types. Patterns with greater facet numbers, which are closer to the facet division sizes of existing conventional shapes and configurations of the 3-D muqarnas patterns, tend to obtain moderately higher scores in Q3 (Beauty), Q5 (Sacred), and Q6 (Preferred) from the participants (i.e., the denser patterns obtained better overall average scores). However, there are no correlations between facet counts and Q1 (Traditional), Q2 (Modernity), or Q3 (Calm, Tranquility). This implies that patterns closely related to traditional muqarnas are more preferred among participants, although some characteristics, such as modernity, are irrespective of facet sizes and counts.

5.4. LIMITATIONS

One limitation of our study is the absence of significant connections found between certain factors governing the geometries, specifically segment numbers and offset parameters used to establish the pattern region for tiling, known as the jump number, and crowdsourced ratings. Further investigation into better parameter selections for geometrical outputs is warranted. Structural performance was estimated based on approximated three-dimensional shapes on the dorm surface, which could be more extensively modeled in future research. Additionally, a more homogeneous projection approach for 2-D to 3-D geometries is required to reduce panel division elongation at the lower half of the dorm shape.

Our analysis yields new insights into how different Muqarnas patterns influence various subjective impressions of designs. While our current focus is on understanding evaluations by participants already familiar with Iranian traditional architecture, expanding these investigations with a larger number of samples and participants would enhance the credibility of our study and allow us to utilize the dataset as training data for various machine learning applications. For example, existing research utilizes such datasets to develop deep neural network models for predicting subjective functionality and comfort from architectural floor plans (Narahara & Yamasaki, 2022; Kitabayashi et al., 2022). Furthermore, there are proposed generative models for architectural designs using data-driven approaches, though these avenues remain as future endeavors.

6. Conclusion

As discussed above, the research aimed to formulate a parametric generative process for new muqarnas patterns. The methodology proposed in this paper could be utilized to generate numerous new designs of muqarnas forms, drawing upon Islamic architectural concepts pertaining to muqarnas patterns and their intrinsic relationship with the aesthetically pleasing nature of muqarnas geometry. The generative design process showcased in this research demonstrates its capability to produce diverse muqarnas patterns while retaining traditional values and aesthetics, ultimately achieving identity-oriented design. The study's conclusion emphasizes that Iranian architects are inclined not only toward novel methodologies for generating muqarnas and employing parametric solutions for creative patterns but also a strong preference for upholding traditional values and proportions in muqarnas pattern design.

References

Ahmed, M., & OLA, F. (2022). Analytical Study of Muqarnas Formations in Islamic Architecture According to Digital Simulation. *Journal of Heritage and Design,* 2(7), 315-342.

Alaçam, S., Güzelci, O. Z., Gürer, E., & Bacınoğlu, S. Z. (2017). Reconnoitering computational potentials of the vault-like forms: Thinking aloud on muqarnas tectonics. *International Journal of Architectural Computing,* 15(4), 285-303.

Alaçam, S., & Güzelci, O. Z. (2016). Computational interpretations of 2-D Muqarnas projections in 3-D form finding.

Dold-Samplonius, Y. (1992). Practical Arabic Mathematics: Measuring the Muqamas by al-Eshi.

Gherardini, F., & Leali, F. (2016). A framework for 3D pattern analysis and reconstruction of Persian architectural elements. *Nexus Network Journal*, 18, 133-167.

Govabari, M., & Ghalamara, S. (2018). Analysis of Muqarnas art in historical periods before and after Islam in Iran and the extent of the spread of this art in the world.[online] Tehran: *Memarishenasi Journal,* Number 5.

Güzelci, O. Z., & Alaçam, S. (2019). A study on measuring complexity in Muqarnas patterns. *Turkish Online Journal of Design Art and Communication,* 9(2), 191-201.

Harmsen, S. (2006). Algorithmic computer reconstructions of stalactite vaults-muqarnas-in Islamic architecture (Doctoral dissertation).

Imani, E. (2017). Historical and geometrical analysis of muqarnas and prospect of its reflection on today's architecture (Master's thesis, Middle East Technical University).

Kitabayashi, R., Narahara, T., & Yamasaki, T. (2022, December). Graph Neural Network Based Living Comfort Prediction Using Real Estate Floor Plan Images. *In Proceedings of the 4th ACM International Conference on Multimedia in Asia* (pp. 1-5). https://doi.org/10.1145/3551626.3564970.

Narahara, T., & Yamasaki, T. (2022). Subjective Functionality and Comfort Prediction for Apartment Floor Plans and Its Application to Intuitive Online Property Search. *IEEE Transactions on Multimedia,* vol. 25. https://doi.org/10.1109/TMM.2022.3214072.

Ödekan, A. (1975). Bir Mukarnaslı Portal Yarım Kubbesi Geometrik Şemadan Üçüncü Boyuta Geçiş Örneği.

Ödekan, A. (1977). Osmanlı öncesi Anadolu Türk mimarisinde Mukarnaslı portal örtüleri. İstanbul: İTÜ Mimarlık Fakültesi Yayınları.

Parakeet3d. (2023). *A collection of DesignTools focusing on Algorithmic Pattern generation.* Retrieved December 7, 2023, from https://www.food4rhino.com/en/app/parakeet.

Preisinger, C. (2013). Linking Structure and Parametric Geometry. *Architectural Design,* 83: 110-113. DOI: 10.1002/ad.1564.

Takahashi, S. (2023). *Muqarnas: A Three-Dimensional Decoration of Islamic Architecture.* Retrieved December 7, 2023, from http://www.shiro1000.jp/muqarnas/default-.htm.

Yaghan, M. A. J. (2001). The Muqarnas pre-designed erecting units: analysis, definition of the generic set of units, and a system of unit-creation as a new evolutionary step. Architectural Science Review, 44(3), 297-318.

Yaghan, M. A. (2005). Self-supporting "genuine" Muqarnas units. Architectural Science Review, 48(3), 245-255.