

# SCALE MISMATCHING: PREVALENT COMPACTNESS INDEXES OF URBAN FORM DO NOT WORK IN COMPUTATIONAL URBAN DESIGN

CHENGYU SUN<sup>1</sup>, YU MENG<sup>2</sup> and XINRU WANG<sup>3</sup>

<sup>1,2,3</sup>*Tongji University; Shanghai Key Laboratory of Urban Renewal and Spatial Optimization Technology*

<sup>1</sup>*ibund@126.com, 0000-0002-5686-5957*

<sup>2</sup>*mengyu@tongji.edu.com, 0009-0007-0905-2885*

<sup>3</sup>*wangx\_ru@126.com, 0009-0004-1316-5581*

**Abstract.** This study systematically evaluates and compares the effectiveness of 9 prevalent indexes for measuring the compactness of building distributions at the meso-scale through comparative experiments. Experiments primarily employ an ideal sample with controlled variables, and superior indicators are further tested in actual urban areas. The assessment, grounded in sensitivity and consistency, adopts rigorous quantitative criteria and is compared against a baseline computed by cohesion evaluation based on graph (GCE). Research findings indicate: (a) when quantifying compactness differences in same scale regions: Directly employing GCE or the improved T\* is recommended; (b) when comparing compactness differences in regions of diverse scales, GCE is recommended; if using ENN or ANN, supplementary evaluation is necessary; (c) in studies of urban morphology effects mechanisms, it is advisable to utilize GCE instead of T\*, as T\* presents collinearity issues with footprint density. None of the remaining indexes is recommended for the above scenario at meso-scale. This research distinctly reveals the limitations of prevalent compactness indexes at meso-scale and suggests superior alternatives.

**Keywords.** urban morphology measurement, building distribution compactness, compactness indexes, index applicability

## 1. Introduction

The compactness indexes under consideration in this study are morphological indexes used to measure the compactness of specific urban elements or types (Lin, 1998). Originating in macro-level urban planning research, these indicators have a longstanding history and matured over time. Initially applied to assess the clustering of land patches, public green spaces, or city facilities, compactness indicators were later introduced to the meso-level architectural research domain for evaluating the compactness of buildings or other elements (Ma and Liu, 2021, Zhuang and Zhou, 2019). The compactness of building distribution aids in automated identification of urban textures (Colaninno et al., 2011) and has been confirmed to significantly impact the microclimate around buildings (Wang et al., 2021). However, there are notable

differences in diverse research scales, leading to variations in the methods used to describe the study objects. Directly applying prevalent macro-scale compactness indexes from urban morphology studies to calculate the compactness of objects at meso-scale in urban design raises questions about their compatibility.

Therefore, this paper aims to point out efficient and superior compactness indexes in computational urban design, specifically at the meso-scale, through the design of ideal samples, comparative experimental analysis and rigorous evaluation criteria. The methods and results of this experiment provide detailed parameters and scientific references for measuring meso-scale building distribution compactness.

## 2. Method

### 2.1. SAMPLE DATA

Actual architectural clusters exhibit significant complexity and diversity (Figure 1a), characterized by multidimensional features. To conduct effective comparative experiments, it is essential to construct ideal samples that abstract these real-world samples, while controlling for other features like regional density or individual differences. This ensures a clear understanding of how indexes values respond to variations in a single feature. Ideal samples design process is detailed as follows:

1. Three sample groups (S200, S500, S1000, with numbers representing area lengths) are set based on common scale parameters in existing research.
2. Using building base data from Gaode Map, we rough estimate the size and density of buildings footprint within the outer ring of Shanghai, China. Adopting the mean area ( $397.6\text{m}^2$ ) and count per hectare (lower quartile = 4) as references, each building is set at  $20\times 20\text{m}$ , with a controlled density of 0.16 for each scale group.
3. As the fundamental characteristic of urban texture (Shen et al., 2021), the proximity distance between buildings ( $d$ ) is taken as the attribute, and 7-level discrete variables are set from 0m to 30m at intervals of 5m.

Additionally, two types of samples are introduced: samples with 5% (6.25% for group S200) outliers (increase irregularity in distribution, with consistent outlier locations in each sample group and neighbor distances  $\geq 30\text{m}$ ), and samples with multiple centers ( $k=4$ , with fixed centroid positions) to test the performance of indexes when further complexity is introduced. Finally, the matrices for the three sample groups are depicted in Figure 2b.

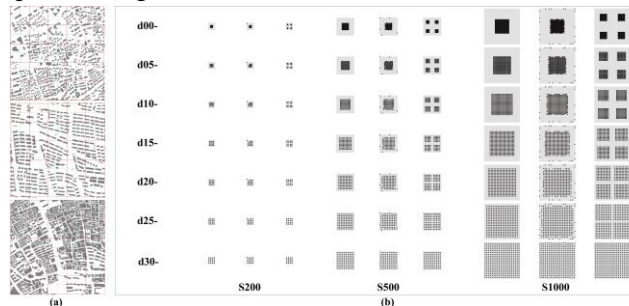


Figure 1. Actual form of buildings in urban and ideal samples

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2.2. PREVALENT COMPACTNESS INDEXES AND CALCULATION

After the review of indexes used in existing studies, this experiment finally selected 9 prevalent compactness indexes for testing:

**1. Circularity compactness Index (COLE)** (Cole, 1964): Measures based on the minimum bounding circle area. This index is widely applied in city-scale studies (Liu and Long, 2021).

**2. Convex Hull Area Compactness Index (CHAD)** (Liu and Long, 2021): Measures based on the convex hull area. It was applied in studies concerning elements buffer zones (Colaninno et al., 2011).

**3. Euclidean Mean Nearest Neighbor Distance (ENN)**: Provides a simple and intuitive representation of spatial distribution based on the centroid distance of neighboring elements. It sees in both urban areas (Dietzel et al., 2005) and city-scale studies (Falihatkar and Rezaei, 2020).

**4. Average Nearest Neighbor Ratio (ANN)**: Measures compactness by comparing the ratio of the average nearest neighbor distance to the expected value. It sees in many community-scale studies (Akrofi and Okitasari, 2023).

**5. Buildings Proximity (PROX)** (Colaninno et al., 2006): Measures compactness based on the proximity of the center to other elements.

**6. Global Moran's I (MI)** (Moran, 1950): A spatial autocorrelation measurement method used to calculate the compactness of spatial layouts based on grid or element attributes. It is widely used at various scales, as cities (Liu et al., 2021), urban areas (Ma and Liu, 2021), and blocks (Rahman et al., 2022).

**7. Degree of Urban Dispersion (DIS)** (Jaeger et al., 2009): Measures the dispersion of distribution by weighted calculation of pixel distances.

**8. Average Gravity Index (T)** (Thin et al., 2002): Introduces Newton's law of universal gravitation to calculate urban form compactness. It could be found in urban areas (Jin et al., 2018) and city-scale (Thin et al., 2002).

**9. Normalized Compact Index (NCI)** (Zhao et al., 2011): As claiming to address the sensitivity issue of the T to the scale of the research object, it has been applied on multiple scales, like city-scale (Jia and Tang, 2019) and buffer zones (Ji et al., 2023).

The diverse application scales of these indicators in existing studies, along with the lack of clarity on specific calculation methods and parameters in some studies, hinder reuse. Here, Table 1 provides detailed descriptions of the calculation formulas for these indexes and the specific calculation methods and parameters in this experiment.

Table 1 Calculation formulas, Tools (in ArcGIS Pro 3.0.2) and parameters of indexes in experiment

Index and calculation formula	Value range	Tools	Parameter
$COLE = \sum a_i / A_c$ <p><math>a_i</math> - area of building <math>i</math> ; <math>A_c</math> - minimum circumscribed circle area of the building group</p>	<p><math>0 &lt;</math></p> <p><math>COLE \leq 1</math></p> <p>Direct ratio</p>	<p><b>Minimum</b></p> <p><b>Bounding</b></p> <p><b>Geometry &amp;</b></p>	<p>Geometry Type:</p> <p><b>Circle</b></p>

$CHAD = \sum a_i / A_{hull}$ <p><math>a_i</math>- area of building <math>i</math>; <math>A_{hull}</math>- minimum convex hull area of the building group</p>	$0 < CHAD \leq 1$ Direct ratio	Field calculation	Geometry Type: <b>Convex hull</b>
$ENN = \sum d_i / n$ <p><math>n</math>- number of buildings  <math>d_i</math>- Euclidean distance between building <math>i</math> and its nearest neighbor</p>	$ENN > 0$ Inverse ratio	<b>Average Nearest Neighbor</b>	Distance Method: <b>Euclidean</b> ; Area: <b>unit area</b>
$ANN = \sum d_i / n / 0.5 / \sqrt{n/A}$ <p><math>n</math>- number of buildings; <math>A</math>-unit area</p>	$ANN > 0$ Inverse ratio		
$PROX = \sum a_i / \sum d^2(i, c) \cdot n$ <p><math>n</math>- number of buildings; <math>a_i</math>- area of building <math>i</math>;  <math>d(i, c)</math>- Euclidean distance between the centers of building <math>i</math> and unit</p>	$PROX > 0$ Direct ratio	Write the formula in python	-
$MI = \frac{n \sum_{i=1}^n \sum_{j=1}^n w_{ij} (X_i - X_j)(X_j - X)}{(\sum_{j=1}^n \sum_{i=1}^n w_{ij}) (X_i - \bar{X})^2}$ <p><math>n</math>- number of grids; <math>X_i, X_j, w_{ij}</math>- building area in grid <math>i</math> and <math>j</math> and the spatial weight between; <math>\bar{X}</math>- expected area</p>	$-1 \leq MI \leq 1$ Direct ratio	<b>Summarize Within &amp; Spatial Autocorrelation</b>	Grid length: <b>10m</b> ; Conceptualization: <b>Inverse distance</b> ; Dis Band: <b>0</b>
$DIS = \frac{1}{n} \sum_{i=1}^n \frac{1}{n_i} (\sum_{j=1}^{n_i} (\sqrt{2d(i, j) + 1} - 1) + C), C = \sqrt{0.97428 b + 1.046} - 0.996249,$ <p><math>n</math>- Total grid number; <math>n_i</math>- grids number within the view scope; <math>d(i, j)</math> - Euclidean distance between grid <math>i</math> and <math>j</math>; <math>b</math>-grid length(m)</p>	$DIS > 0$ Inverse ratio	Write the formula in python	$n$ & $n_i$ : <b>number of all grids in unit</b> ; $b$ : <b>20m</b>
$T = \sum \frac{X_i X_j}{d^2(i, j)} / n(n-1)/2$ <p><math>n</math>- grids number; <math>X_i, X_j, d(i, j)</math>- building area in grid <math>i</math> and <math>j</math> and the distance between</p>	$T > 0$ Direct ratio	<b>Summarize Within &amp;</b> Write the formula in python	Grid length: <b>10m</b> $n$ : <b>number of all grids in unit</b> ; $c$ : <b>1</b>
$NCI = T / T_{max}$ <p><math>T_{max}</math>-T value computed by the equivalent circle</p>	$0 < NCI \leq 1$ Direct ratio		

### 2.3. INDEX EVALUATION CRITERIA: SENSITIVITY & CONSISTENCY

The ideal indexes should be use-easy, with moderate computational requirements and devoid of complex tuning processes. Furthermore, for the meso-scale, the accuracy of results should fall within an appropriate range. To address the question of evaluating the appropriateness of result accuracy, this experiment establishes sensitivity and consistency criteria and provides a quantitative method for assessing the effectiveness of indexes calculations.

Sensitivity criteria are employed to detect whether indicators exhibit abnormal responses to different compactness distributions within the same scale. This is categorized into C1-Sensitivity[S], C2-Sensitivity[O], and C3-Sensitivity[M] for different aggregation scenarios. The detailed assessment items are as Table 2.

Consistency criteria are to examine whether indexes demonstrate consistent response values to the same compactness distribution at different area scales, or if

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differences are acceptable. Specific methods include diff analysis (calculating the sum of squared differences per two groups (SSD) and the coefficient of determination( $R^2$ )), rank sum tests per two groups, and the Kruskal-Wallis H test for all three groups.

Additionally, drawing from the concept of cluster evaluation in data mining (Tan et al., 2016), we apply the cohesion evaluation based on graph (GCE, Formula 1, where  $a_i a_j$  represent the area product of buildings i and j within pairs of grids in the region,  $d(i, j)$  is the centroid distance, and  $C$  is a constant to prevent overly small values, set to 10 here) and measure its sensitivity. It serves as a baseline for consistency evaluation. If other indexes do not meet this baseline, researchers are advised to directly employ this baseline formula for compactness measurements.

Table 2. Detailed assessment items and interpretation of sensitivity criteria

### C1. Sensitivity [S]

- c1. Any S-group is monotone (if not: the measurement results are not credible)
- c2. The differences among values within a S-group is all over 0.005 (if not: nearest neighbors distance differences are hardly perceptible under the parameters of this experiment)

### C2. Sensitivity [O]

- c3. Any O-group is monotone (if not: the measurement results are not credible)
- c4. The differences among values within a O-group is all over 0.005 (if not: nearest neighbors distance differences are hardly perceptible under the parameters of this experiment)
- c5. The difference between group S&O is one-sided (if not: the presence of outliers leads to measurement errors)
- c6. The difference between group S&O is all over 0.005 (if not: outliers are hardly perceptible under the parameters of this experiment)
- c7. The difference between the groups S&O is below 15% (if not: 5% outliers result in a numerical difference of more than 3 times)

### C3. Sensitivity [M]

- c8. Any M-group is monotone (if not: the measurement results are not credible)
- c9. The difference among values within a M-group is all over 0.005 (if not: nearest neighbors distance differences are hardly perceptible under the parameters of this experiment)
- c10. The difference between group S&M is one-sided (if not: the presence of multi-centre feature leads to measurement errors)
- c11. The difference between group S&M is all over 0.005 (if not: multi-centre feature is hardly perceptible under the parameters of this experiment)

$$baseline(GCE) = \frac{\sum_{i \in C, j \in C} a_i a_j d(i, j)}{\sum_{i \in C, j \in C} a_i a_j} * \frac{C}{unit\ length} \quad (1)$$

## 3. Result and Discuss

Results of indexes values calculated for all samples see Figure 2.

### 3.1. ABNORMALITIES IN SENSITIVITY

In urban design, comparing the morphological conditions before and after the design of a particular area or the differences between different design schemes is a common

analytical scenario. However, not all commonly used compactness indices prove effective in these situations. Computed results are scrutinized based on the sensitivity criteria one by one, and the failure items detected were summarized in Table 3.

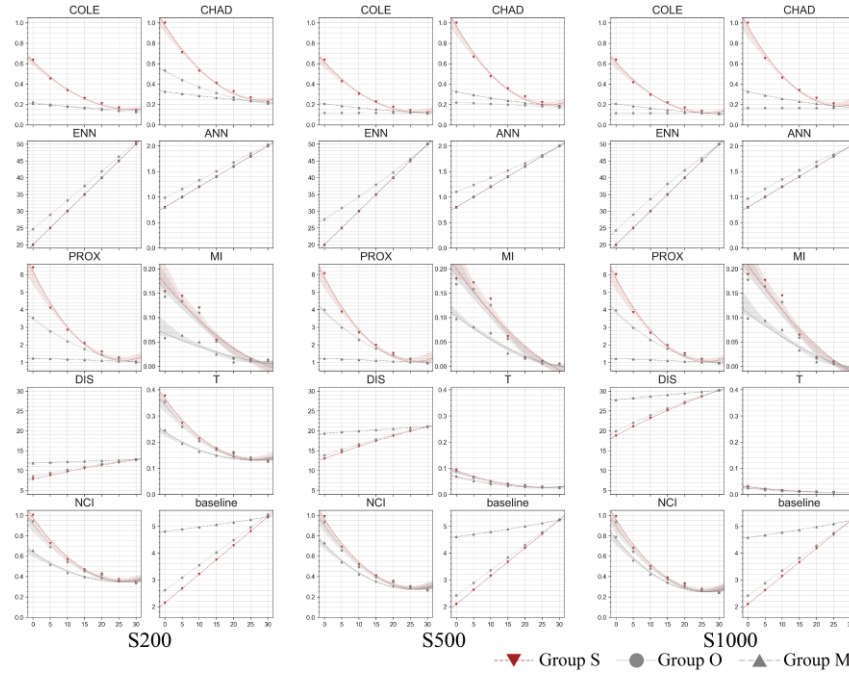


Figure 2 Indexes calculation results (quadratic polynomial regression, 50% confidence interval)

Table 3 Results of index sensitivity evaluation

Criteria	Index	Index										
		ENN	ANN	PROX	CHAD	COLE	T	T*	NCI	MI	DIS	baseline
C1	c1											
	c2						×				×	
	c3											
C2	c4				×	×	×				×	
	c5										×	
	c6		×	×		×	×	×	×	×		
	c7			×	×	×						
C3	c8										×	
	c9						×				×	
	c10											
	c11	×	×				×				×	

c7 judgements is according to red values in Table 4.

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Table 4 Analysis of differences between values from group S&O

	ENN	ANN	PROX	CHAD	COLE	T	T*	NCI	MI	DIS	baseline
<b>S200</b>	9.4%	9.4%	19.6%	25.1%	38.2%	3.3%	3.3%	3.3%	7.8%	4.0%	9.2%
<b>S500</b>	12.9%	12.9%	13.1%	43.2%	45.2%	3.3%	3.3%	3.3%	7.0%	2.6%	5.9%
<b>S1000</b>	9.4%	9.4%	12.9%	47.9%	44.2%	3.4%	3.4%	3.4%	7.7%	2.5%	5.8%
<b>all</b>	10.6%	10.6%	15.2%	38.7%	42.5%	3.3%	3.3%	3.3%	7.5%	3.0%	6.9%

the Ratio is the mean of the ratio of every diff of pair values from group S & O and value from group O.

Evidently, the baseline demonstrates excellent sensitivity in the experiment. Additionally, a tested multi-scale comparison using  $T/area$  (Jin et al., 2018), referred to as  $T^*$ , showed a significant improvement in sensitivity. While DIS exhibits good sensitivity in this experiment, its binary raster-based calculation method may lack precision with diverse individual building morphologies. In summary, when comparing building distribution differences within the same scale, GCE method or the improved  $T^*$  are suitable choices.

### 3.2. ABNORMALITIES IN CONSISTENCY

In urban design comparisons involving multiple cases, there often exist scale differences. Haphazard applying compactness indexes for comparative calculations can pose significant issues. According to consistency analysis results (Table 5),

- From the results of rank sum tests, ENN and ANN exhibit the best consistency, while PROX, CHAD, COLE, MI, and NCI show consistent differences, though not statistically significant. DIS and T display significant consistency anomalies (Table 5, red values). In contrast to the claims in existing studies, the consistency of  $T^*$  does not significantly improve over T, still exhibiting notable differences.

Table 5 Results of index consistency evaluation

Index	Diff Analysis (z-score normalization)						Rank Sum Test			K-W H Test
	S200, S500		S500, S1000		S200, S1000		S200, S500	S200, S1000	S500, S1000	All group
	SSD	R <sup>2</sup>	SSD	R <sup>2</sup>	SSD	R <sup>2</sup>				
ENN	0.000	1.000	0.000	1.000	0.000	1.000	-	-	-	-
ANN	0.000	1.000	0.000	1.000	0.000	1.000	-	-	-	-
PROX	0.071	0.990	0.001	1.000	0.092	0.987	-	-	-	-
CHAD	0.194	0.972	0.015	0.998	0.318	0.955	-	-	-	-
COLE	0.193	0.972	0.015	0.998	0.317	0.955	-	-	-	-
MI	0.444	0.937	0.030	0.996	0.689	0.903	-	-	-	-
NCI	0.424	0.939	0.046	0.993	0.732	0.895	-	-	-	-
DIS	20.139	-1.877	24.889	-1.333	24.578	-2.511	***	***	***	***
T	20.226	-1.911	16.102	-1.360	23.716	-2.372	***	***	***	***
T*	24.183	-2.455	22.324	-2.189	22.963	-2.455	***	***	***	***

baseline	0.044	0.994	0.001	1.000	0.059	0.991	-	-	-	-
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The higher the value of  $R^2$ , the better the consistency

- Although PROX, CHAD, COLE, MI, and NCI show no significant differences in rank sum tests, their performance is not flawless (especially NCI, a cross-scale consistency indicator proposed in existing studies, exhibits decreasing values with increasing regional scales). Examining the difference analysis results, their consistency performance falls short of the baseline.

Therefore, when analyzing the differences in the compactness of distributions across scales for multiple cases, we recommend directly using GCE. ENN and ANN should be used in conjunction with other indexes together.

### 3.3. CASE STUDY: VERIFICATION AND COMPARISON OF BETTER INDEXES

To demonstrate the effectiveness and distinctions of the identified indices, practical calculations were conducted in the Jing'an District of Shanghai.  $T^*$  and GCE were calculated based on S200, S500, and S1000 (Figure 3, a). Due to space limitations, visual results for  $T^*$  at S500 and GCE at S500 and S1000 are presented (Figure 3, b-d,  $T^*$  use k-medoids clustering for classification, GCE using equal interval) to illustrate the effectiveness and distinctions.

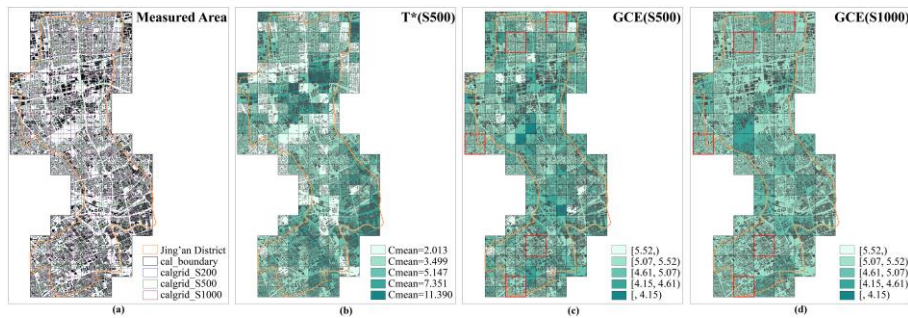


Figure 3 The measured effect of  $T^*$  and GCE in actual city area

Based on Figure 3b and c,  $T^*$  seems more capable of distinguishing different urban textures. We analyze the reasons for this phenomenon. In the sample experiment, we controlled the building density within regions, while in the actual measurements, these grids had varying building densities. Using the calculated values based on S200 (as the sample size was sufficient), Spearman and Pearson correlation tests were employed to analyze the correlation between  $T^*$ , GCE, and footprint density (Table 6). The results indicate a strong correlation between  $T^*$  and footprint density, while the correlation between GCE and footprint density is weak. In studies on the mechanisms of urban morphology effects, compactness and footprint density (or GSI) are often both included as independent variables in the fitting process. In such cases, simultaneous use of  $T^*$  and footprint density introduces a significant issue of collinearity, whereas GCE demonstrate superior effective.

Furthermore, GCE exhibits better consistency in actual measurements. Based on



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Figure 3c and d, in several areas with approximate and uniform distributions (highlighted in red boxes), the values of the regions are essentially the same.

Table 6 Correlation coefficient matrix for T\*, GCE and footprint density

	Spearman Correlation Test			Pearson Correlation Test		
	fp density	T*	GCE	fp density	T*	GCE
fp density	—	0.955***	0.219***	—	0.929***	0.367***
T*		—	0.102***		—	0.183***
GCE			—			—

**4. Conclusion**

This study systematically evaluated the applicability of compactness indexes for building distribution, providing clear reference for the appropriate measurements in different research scenarios. The key conclusions are as follows:

- When quantifying compactness differences in same scale regions: Directly employing GCE method or the improved T\* is recommended.
- When quantifying compactness differences in regions of diverse scales: Directly using GCE is recommended. If using ENN and ANN, it is necessary to evaluate them in conjunction with other indexes.
- In studies of urban morphology effect mechanisms: It is advised to use GCE instead of T\* because of collinearity issues with footprint density, potentially not work.

The study emphasizes the dependence of urban morphology measurement indices on application scenarios, research scales, and studied objects. A careful consideration of index calculation methods and an assessment of their applicability are necessary. Researchers should enhance their understanding of calculation formula, clarifying the principles and parameters of each measurement.

Additionally, this study, rooted in interdisciplinary thinking, explored a more effective compactness measurement method, GCE, and provided detailed explanations. The results demonstrate the effectiveness of interdisciplinary exploration.

As urban fine-grained governance continues to advance, the demand for high-resolution urban morphology measurement increases. For meso- and micro-scale morphology measurement, the potential of morphology characterization systems based on regional geometric statistics appears limited. Therefore, we suggest that the future direction should involve the development of more accurate and comprehensive morphology characterization systems, integrating interdisciplinary methods.

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